

Charmed $(70, 1^-)$ baryon multiplet

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Abstract

The masses of negative parity $(70, 1^-)$ charmed nonstrange baryons are calculated in the relativistic quark model. The relativistic three-quark equations of the $(70, 1^-)$ charmed baryon multiplet are found in the framework of the dispersion relation technique. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. The calculated mass values of the $(70, 1^-)$ charmed baryons are in good agreement with the experimental data.

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PACS: 11.55.Fv, 12.39.Ki, 12.40.Yx, 14.20.Lq.

1. Introduction.

For many years CLEO was the main source of data on orbitally-excited charmed baryons [1]. An excited Σ_c candidate has been seen decaying to $\Lambda_c \pi^+$, with mass about 510 MeV above $M(\Lambda_c)$ [2]. The first excitation of the Λ_c and Ξ_c scale well from the first Λ excitations $\Lambda(1405, \frac{1}{2}^-)$ and $\Lambda(1520, \frac{3}{2}^-)$. The highest Λ_c was seen by BaBar in decay mode $D^0 p$ [3]. The highest Ξ_c were reported by the Belle Collaboration in Ref. [4] and confirmed by BaBar [5].

In the recent reviews [6, 7] the spectroscopy of hadrons containing heavy quarks and some of their theoretical interpretation are given. One discuss progress on orbitally excited charmed baryons.

In the series of papers [8 – 12] a practical treatment of relativistic three-hadron systems have been developed. The physics of three-hadron system is usefully described in term of pairwise interactions among the three particles. The theory is based on the two principles of unitarity and analyticity, as applied to the two-body subenergy channels. The linear integral equation in a single variable are obtained for the isobar amplitudes.

Instead of the quadrature methods of obtaining solution the set of suitable functions is identified and used as basis set for the expansion of the desired solutions. By this means the couple integral equation are solved in terms of simple algebra.

In our papers [13, 14] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting particles. The mass spectrum of S -wave baryons including u , d , s -quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such an approximation, which corresponds to taking into account two-body and triangle singularities, defined all the smooth functions of the subenergy

variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the recent paper [15] the relativistic three-quark equations of the excited $(70, 1^-)$ baryons are found in the framework of the dispersion relation technique. In our paper the orbital-spin-flavor wave functions are constructed. We have used the orbital-spin-flavor wave functions for the construction of integral equations. We take into account the u , d , s -quarks. We have represented the 30 nonstrange and strange resonances belonging to the $(70, 1^-)$ multiplet. The 15 resonances are in good agreement with experimental data. We have predicted 15 masses of baryons. In our model the four parameters are used: gluon coupling constants g_+ and g_- for the various parity, cutoff energy parameters λ , λ_s for the nonstrange and strange diquarks.

The present paper is organized as follows. Section 2 is devoted to the construction of the orbital-spin-flavor wave functions for the charmed baryons $(70, 1^-)$ multiplet. In Section 3 the relativistic three-quark equations are constructed in the form of the dispersion relation over the two-body subenergy. In Section 4 the systems of equations for the reduced amplitudes are derived. Section 5 is devoted to the calculation results for the P -wave charmed baryons mass spectrum (Tables I-IV). In Conclusion, the status of the considered model is discussed.

2. The wave function of $(70, 1^-)$ excited charmed states.

Here we deal with a three-quark system having one unit of orbital excitation. We take into account u , d , c -quarks. The orbital part of wave function must have a mixed symmetry. The spin-flavor part of wave function possesses the same symmetry in order to obtain a totally symmetric state in the orbital-spin-flavor space.

For the sake of simplicity we derived the wave functions for the decuplets $(10, 2)$. The fully symmetric wave function for the decuplet state can be constructed as [16].

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right). \quad (1)$$

Then we obtain:

$$\varphi = \frac{1}{\sqrt{2}} \varphi_S^{SU(3)} \left(\varphi_{MA}^{SU(2)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(2)} \varphi_{MS}^{O(3)} \right), \quad (2)$$

here MA and MS define the mixed antisymmetric and symmetric part of wave function,

$$\varphi_{MA}^{SU(6)} = \varphi_S^{SU(3)} \varphi_{MA}^{SU(2)}, \quad \varphi_{MS}^{SU(6)} = \varphi_S^{SU(3)} \varphi_{MS}^{SU(2)}. \quad (3)$$

The functions $\varphi_{MA}^{SU(2)}$, $\varphi_{MS}^{SU(2)}$, $\varphi_{MA}^{O(3)}$, $\varphi_{MS}^{O(3)}$ are given:

$$\varphi_{MA}^{SU(2)} = \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \varphi_{MS}^{SU(2)} = \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow), \quad (4)$$

$$\varphi_{MA}^{O(3)} = \frac{1}{\sqrt{2}} (010 - 100), \quad \varphi_{MS}^{O(3)} = \frac{1}{\sqrt{6}} (010 + 100 - 2 \cdot 001). \quad (5)$$

\uparrow and \downarrow determine the spin directions. 1 and 0 correspond to the excited or nonexcited quarks. The three projections of orbital angular momentum are $l_z = 1, 0, -1$. The $(10, 2)$ multiplet with $J^P = \frac{3}{2}^-$ can be obtained using the spin $S = \frac{1}{2}$ and $l_z = 1$, but the $(10, 2)$ multiplet with $J^P = \frac{1}{2}^-$ is determined by the spin $S = \frac{1}{2}$ and $l_z = 0$.

We construct the $SU(3)$ -function for each particle of multiplet. For instance, the $SU(3)$ -function for Σ_c^+ -hyperon of decuplet have following form:

$$\varphi_S^{SU(3)} = \frac{1}{\sqrt{3}} (ucu + cuu + uuc). \quad (6)$$

We obtain the $SU(6) \times O(3)$ -function for the Σ_c^+ of the $(10, 2)$ multiplet:

$$\begin{aligned} \varphi_{\Sigma_c^+(10,2)} = & \frac{\sqrt{6}}{18} \left(2\{u^1 \downarrow u \uparrow c \uparrow\} + \{c^1 \downarrow u \uparrow u \uparrow\} - \right. \\ & \left. - \{u^1 \uparrow u \downarrow c \uparrow\} - \{u^1 \uparrow u \uparrow c \downarrow\} - \{c^1 \uparrow u \uparrow u \downarrow\} \right). \end{aligned} \quad (7)$$

Here the parenthesis determine the symmetrical function:

$$\{abc\} \equiv abc + acb + bac + cab + bca + cba. \quad (8)$$

The wave functions of Σ_c^0 - and Σ_c^- -hyperons can be constructed by similar way.

For the $\Xi_{cc}^{0,-}$ state of the $(10, 2)$ multiplet the wave function is similar to the $\Sigma_c^{+,-}$ state with the replacement by $u \leftrightarrow c$ or $d \leftrightarrow c$. The wave function for the Ω_{ccc} of $(10, 2)$ decuplet is determined as:

$$\varphi_{\Omega_{ccc}(10,2)} = \frac{\sqrt{2}}{6} \left(\{c^1 \downarrow c \uparrow c \uparrow\} - \{c^1 \uparrow c \uparrow c \downarrow\} \right). \quad (9)$$

The wave functions and the method of the construction for the multiplets $(8, 2)$, $(8, 4)$ and $(1, 2)$ are similar.

3. The three-quark integral equations for the $(70, 1^-)$ multiplet.

By the construction of $(70, 1^-)$ charmed baryon multiplet integral equations we need to using the projectors for the different diquark states. The projectors to the symmetric and antisymmetric states can be obtained as:

$$\frac{1}{2} (q_1 q_2 + q_2 q_1), \quad \frac{1}{2} (q_1 q_2 - q_2 q_1). \quad (10)$$

$$\frac{1}{2} (\uparrow\downarrow + \downarrow\uparrow), \quad \frac{1}{2} (\uparrow\downarrow - \downarrow\uparrow). \quad (11)$$

$$\frac{1}{2} (10 + 01), \quad \frac{1}{2} (10 - 01). \quad (12)$$

One can obtain the four types of totally symmetric projectors:

$$S = S \cdot S \cdot S = \frac{1}{8} (q_1 q_2 + q_2 q_1) (\uparrow\downarrow + \downarrow\uparrow) (10 + 01), \quad (13)$$

$$S = S \cdot A \cdot A = \frac{1}{8} (q_1 q_2 + q_2 q_1) (\uparrow\downarrow - \downarrow\uparrow) (10 - 01), \quad (14)$$

$$S = A \cdot A \cdot S = \frac{1}{8} (q_1 q_2 - q_2 q_1) (\uparrow\downarrow - \downarrow\uparrow) (10 + 01), \quad (15)$$

$$S = A \cdot S \cdot A = \frac{1}{8} (q_1 q_2 - q_2 q_1) (\uparrow\downarrow + \downarrow\uparrow) (10 - 01). \quad (16)$$

We use these projectors for the consideration of various diquarks:

$u^1 \uparrow c \downarrow :$

$$\begin{aligned}
& \frac{A_1^{0c}}{8} (u^1 \uparrow c \downarrow + u^1 \downarrow c \uparrow + c^1 \uparrow u \downarrow + c^1 \downarrow u \uparrow + u \uparrow c^1 \downarrow + u \downarrow c^1 \uparrow + c \uparrow u^1 \downarrow + c \downarrow u^1 \uparrow) \\
& + \frac{A_0^{1c}}{8} (u^1 \uparrow c \downarrow - u^1 \downarrow c \uparrow + c^1 \uparrow u \downarrow - c^1 \downarrow u \uparrow - u \uparrow c^1 \downarrow + u \downarrow c^1 \uparrow - c \uparrow u^1 \downarrow + c \downarrow u^1 \uparrow) \\
& + \frac{A_0^{0c}}{8} (u^1 \uparrow c \downarrow - u^1 \downarrow c \uparrow - c^1 \uparrow u \downarrow + c^1 \downarrow u \uparrow + u \uparrow c^1 \downarrow - u \downarrow c^1 \uparrow - c \uparrow u^1 \downarrow + c \downarrow u^1 \uparrow) \\
& + \frac{A_1^{1c}}{8} (u^1 \uparrow c \downarrow + u^1 \downarrow c \uparrow - c^1 \uparrow u \downarrow - c^1 \downarrow u \uparrow - u \uparrow c^1 \downarrow - u \downarrow c^1 \uparrow + c \uparrow u^1 \downarrow + c \downarrow u^1 \uparrow),
\end{aligned} \tag{17}$$

$u^1 \uparrow c \uparrow :$

$$\begin{aligned}
& \frac{A_1^{0c}}{4} (u^1 \uparrow c \uparrow + c^1 \uparrow u \uparrow + u \uparrow c^1 \uparrow + c \uparrow u^1 \uparrow) + \\
& + \frac{A_1^{1c}}{4} (u^1 \uparrow c \uparrow - c^1 \uparrow u \uparrow - u \uparrow c^1 \uparrow + c \uparrow u^1 \uparrow),
\end{aligned} \tag{18}$$

$u \uparrow c \downarrow :$

$$\begin{aligned}
& \frac{A_1^{0c}}{4} (u \uparrow c \downarrow + u \downarrow c \uparrow + c \uparrow u \downarrow + c \downarrow u \uparrow) + \\
& + \frac{A_0^{0c}}{4} (u \uparrow c \downarrow - u \downarrow c \uparrow - c \uparrow u \downarrow + c \downarrow u \uparrow),
\end{aligned} \tag{19}$$

$u \uparrow c \uparrow :$

$$\frac{A_1^{0c}}{2} (u \uparrow c \uparrow + c \uparrow u \uparrow). \tag{20}$$

Here the lower index determines the value of spin projection, and the upper index corresponds to the value of orbital angular momentum.

We consider the projectors (21)-(24), which are similar to (17)-(20) with the replacement by $c \rightarrow u$ and use the amplitudes A_1^0 , A_0^1 , A_0^0 , A_1^1 . The A is the three-quark amplitude.

$u^1 \uparrow u \downarrow :$

$$\begin{aligned}
& \frac{A_1^0}{4} (u^1 \uparrow u \downarrow + u^1 \downarrow u \uparrow + u \uparrow u^1 \downarrow + u \downarrow u^1 \uparrow) + \\
& + \frac{A_0^1}{4} (u^1 \uparrow u \downarrow - u^1 \downarrow u \uparrow - u \uparrow u^1 \downarrow + u \downarrow u^1 \uparrow),
\end{aligned} \tag{21}$$

$u^1 \uparrow u \uparrow :$

$$\frac{A_1^0}{2} (u^1 \uparrow u \uparrow + u \uparrow u^1 \uparrow), \tag{22}$$

$u \uparrow u \downarrow :$

$$\frac{A_1^0}{2} (u \uparrow u \downarrow + u \downarrow u \uparrow), \quad (23)$$

$u \uparrow u \uparrow :$

$$A_1^0 u \uparrow u \uparrow. \quad (24)$$

Here we consider the projection of orbital angular momentum $l_z = +1$. We use only diquarks $1^+, 0^+, 2^-, 1^-$. If we consider the $l_z = -1$ or $l_z = 0$, that we obtain the other diquarks: $1^+, 0^+, 1^-, 0^-$. In our model the five types of diquarks $1^+, 0^+, 2^-, 1^-, 0^-$ are constructed.

For the sake of simplicity we derive the relativistic Faddeev equations using the Σ_c hyperon with $J^P = \frac{3}{2}^-$ of the (10,2) multiplets. We use the graphic equations for the amplitudes $A_J(s, s_{ik})$. In order to represent the amplitude $A_J(s, s_{ik})$ in the form of dispersion relations, it is necessary to define the amplitudes of quark-quark interaction $a_J(s_{ik})$. The pair quarks amplitudes $qq \rightarrow qq$ are calculated in the framework of the dispersion N/D method with the input four-fermion interaction with quantum numbers of the gluon [17]. We use results of our relativistic quark model [18] and write down the pair quark amplitudes in the form:

$$a_J(s_{ik}) = \frac{G_J^2(s_{ik})}{1 - B_J(s_{ik})}, \quad (25)$$

$$B_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\infty} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J^2(s'_{ik})}{s'_{ik} - s_{ik}}, \quad (26)$$

$$\begin{aligned} \rho_J(s_{ik}) = & \frac{(m_i + m_k)^2}{4\pi} \left(\alpha_J \frac{s_{ik}}{(m_i + m_k)^2} + \beta_J + \frac{\delta_J}{s_{ik}} \right) \times \\ & \times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}. \end{aligned} \quad (27)$$

Here G_J is the diquark vertex function; $B_J(s_{ik})$, $\rho_J(s_{ik})$ are the Chew-Mandelstam function [19] and the phase space consequently. s_{ik} is the two-particle subenergy squared (i,k=1,2,3), s is the systems total energy squared. For the state $J^P = \frac{3}{2}^-$ of the (10,2) multiplet we use three diquarks $J^P = 1^+, 1_c^+, 1_c^-$. The coefficients of Chew-Mandelstam function α_J , β_J and δ_J are given in Table V.

In the case in question the interacting quarks do not produce bound state, then the integration in dispersion integrals is carried out from $(m_i + m_k)^2$ to ∞ .

All diagrams are classified over the last quark pair (Fig.1).

We use the diquark projectors. We consider the particle $\Sigma_c \frac{3}{2}^-$ of the (10,2) multiplet again. This wave function contains the contribution $u^1 \downarrow u \uparrow c \uparrow$, which includes three diquarks: $u^1 \downarrow u \uparrow$, $u^1 \downarrow c \uparrow$ and $u \uparrow c \uparrow$. The diquark projectors allow us to obtain the equations (28)-(30) (with the definition $\rho_J(s_{ij}) \equiv k_{ij}$).

$$\begin{aligned} k_{12} \left(\frac{A_1^0 + A_0^1}{4} (u^1 \downarrow u \uparrow c \uparrow + u \uparrow u^1 \downarrow c \uparrow) + \right. \\ \left. + \frac{A_1^0 - A_0^1}{4} (u^1 \uparrow u \downarrow c \uparrow + u \downarrow u^1 \uparrow c \uparrow) \right), \end{aligned} \quad (28)$$

$$\begin{aligned}
& k_{13} \left(\frac{A_1^{0c} + A_0^{1c} + A_0^{0c} + A_1^{1c}}{8} (u^1 \downarrow u \uparrow c \uparrow + c \uparrow u \uparrow u^1 \downarrow) + \right. \\
& + \frac{A_1^{0c} - A_0^{1c} - A_0^{0c} + A_1^{1c}}{8} (u^1 \uparrow u \uparrow c \downarrow + c \downarrow u \uparrow u^1 \uparrow) + \\
& + \frac{A_1^{0c} + A_0^{1c} - A_0^{0c} - A_1^{1c}}{8} (c^1 \downarrow u \uparrow u \uparrow + u \uparrow u \uparrow c^1 \downarrow) + \\
& \left. + \frac{A_1^{0c} - A_0^{1c} + A_0^{0c} - A_1^{1c}}{8} (c^1 \uparrow u \uparrow u \downarrow + u \downarrow u \uparrow c^1 \uparrow) \right), \quad (29)
\end{aligned}$$

$$k_{23} \left(\frac{A_1^{0c}}{2} (u^1 \downarrow u \uparrow c \uparrow + u^1 \downarrow c \uparrow u \uparrow) \right). \quad (30)$$

Then all members of wave function can be considered. After the grouping of these members we can obtain:

$$u^1 \downarrow u \uparrow c \uparrow \left\{ k_{12} \frac{A_1^0 + 3A_0^1}{4} + k_{13} \frac{A_1^{0c} + 3A_0^{1c}}{4} + k_{23} A_1^{0c} \right\}. \quad (31)$$

The left side of the diagram (Fig.2) corresponds to the quark interactions. The right side of Fig.2 determines the zero approximation (first diagram) and the subsequent pair interactions (second diagram). The contribution to $u^1 \downarrow u \uparrow c \uparrow$ is shown in the Fig.3.

If we group the same members we obtain the system integral equations for the Σ_c state with the $J^p = \frac{3}{2}^-$ (10, 2) multiplet:

$$\left\{ \begin{aligned}
& A_1^0(s, s_{12}) = \lambda b_{1+}(s_{12}) L_{1+}(s_{12}) + K_{1+}(s_{12}) \left[\frac{1}{4} A_1^{0c}(s, s_{13}) + \frac{3}{4} A_0^{1c}(s, s_{13}) + \right. \\
& \quad \left. + \frac{1}{4} A_1^{0c}(s, s_{23}) + \frac{3}{4} A_0^{1c}(s, s_{23}) \right] \\
& A_1^{0c}(s, s_{13}) = \lambda b_{1c^+}(s_{13}) L_{1c^+}(s_{13}) + K_{1c^+}(s_{13}) \left[\frac{1}{2} A_1^0(s, s_{12}) - \frac{1}{4} A_1^{0c}(s, s_{12}) + \right. \\
& \quad \left. + \frac{3}{4} A_0^{1c}(s, s_{12}) + \frac{1}{2} A_1^0(s, s_{23}) - \frac{1}{4} A_1^{0c}(s, s_{23}) + \frac{3}{4} A_0^{1c}(s, s_{23}) \right] \\
& A_1^{1c}(s, s_{23}) = \lambda b_{1c^-}(s_{23}) L_{1c^-}(s_{23}) + K_{1c^-}(s_{23}) \left[\frac{1}{2} A_1^0(s, s_{12}) + \frac{1}{4} A_1^{0c}(s, s_{12}) + \right. \\
& \quad \left. + \frac{1}{4} A_0^{1c}(s, s_{12}) + \frac{1}{2} A_1^0(s, s_{13}) + \frac{1}{4} A_1^{0c}(s, s_{13}) + \frac{1}{4} A_0^{1c}(s, s_{13}) \right].
\end{aligned} \right. \quad (32)$$

Here function $L_J(s_{ik})$ has the form

$$L_J(s_{ik}) = \frac{G_J(s_{ik})}{1 - B_J(s_{ik})}. \quad (33)$$

The integral operator $K_J(s_{ik})$ is:

$$K_J(s_{ik}) = L_J(s_{ik}) \int_{(m_i+m_k)^2}^{\infty} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik}) G_J(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^1 \frac{dz}{2}. \quad (34)$$

$$b_J(s_{ik}) = \int_{(m_i+m_k)^2}^{\infty} \frac{ds'_{ik}}{\pi} \frac{\rho_J(s'_{ik})G_J(s'_{ik})}{s'_{ik} - s_{ik}}. \quad (35)$$

The function $b_J(s_{ik})$ is the truncated function of Chew-Mandelstam. z is the cosine of the angle between the relative momentum of particles i and k in the intermediate state and the momentum of particle j in the final state, taken in the c.m. of the particles i and k . Let some current produces three quarks (first diagram Fig.1) with the vertex constant λ . This constant do not affect to the spectra mass of excited baryons.

By analogy with the $\Sigma_c \frac{3}{2}^- (10, 2)$ state we obtain the rescattering amplitudes of the three various quarks for all P -wave states of the $(70, 1^-)$ multiplet which satisfy the system of integral equations.

4. The reduced equations of $(70, 1^-)$ multiplet.

Let us extract two-particle singularities in $A_J(s, s_{ik})$:

$$A_J(s, s_{ik}) = \frac{\alpha_J(s, s_{ik})b_J(s_{ik})G_J(s_{ik})}{1 - B_J(s_{ik})}, \quad (36)$$

$\alpha_J(s, s_{ik})$ is the reduced amplitude. Accordingly all integral equations can be rewritten using the reduced amplitudes. For instance, one consider the first equation of system for the $\Sigma_c J^p = \frac{3}{2}^-$ of the $(10, 2)$ multiplet:

$$\begin{aligned} \alpha_1^0(s, s_{12}) = & \lambda + \frac{1}{b_{1+}(s_{12})} \int_{(m_1+m_2)^2}^{\Lambda_{1+}(1,2)} \frac{ds'_{12}}{\pi} \frac{\rho_{1+}(s'_{12})G_{1+}(s'_{12})}{s'_{12} - s_{12}} \times \\ & \times \int_{-1}^1 \frac{dz}{2} \left(\frac{G_{1c}^+(s'_{13})b_{1c}^+(s'_{13})}{1 - B_{1c}^+(s'_{13})} \frac{1}{2} \alpha_1^{0c}(s, s'_{13}) + \frac{G_{1c}^-(s'_{13})b_{1c}^-(s'_{13})}{1 - B_{1c}^-(s'_{13})} \frac{3}{2} \alpha_0^{1c}(s, s'_{13}) \right). \end{aligned} \quad (37)$$

The connection between s'_{12} and s'_{13} is [20]:

$$\begin{aligned} s'_{13} = & m_1^2 + m_3^2 - \frac{(s'_{12} + m_3^2 - s)(s'_{12} + m_1^2 - m_2^2)}{2s'_{12}} \pm \\ & \pm \frac{z}{2s'_{12}} \times \sqrt{(s'_{12} - (m_1 + m_2)^2)(s'_{12} - (m_1 - m_2)^2)} \times \\ & \times \sqrt{(s'_{12} - (\sqrt{s} + m_3)^2)(s'_{12} - (\sqrt{s} - m_3)^2)}. \end{aligned} \quad (38)$$

The formula for s'_{23} is similar to (38) with z replaced by $-z$. Thus $A_1^{0c}(s, s'_{13}) + A_1^{0c}(s, s'_{23})$ must be replaced by $2A_1^{0c}(s, s'_{13})$. $\Lambda_J(i, k)$ is the cutoff at the large value of s_{ik} , which determines the contribution from small distances.

The construction of the approximate solution of the (37) is based on the extraction of the leading singularities which are close to the region $s_{ik} = (m_i + m_k)^2$ [20]. Amplitudes with different number of rescattering have the following structure of singularities. The main singularities in s_{ik} are from pair rescattering of the particles i and k . First of all there are threshold square root singularities. Also possible are pole singularities,

which correspond to the bound states. The diagrams in Fig.2 apart from two-particle singularities have their own specific triangle singularities. Such classification allows us to search the approximate solution of (37) by taking into account some definite number of leading singularities and neglecting all the weaker ones.

We consider the approximation, which corresponds to the single interaction of all three particles (two-particle and triangle singularities) and neglecting all the weaker ones.

The functions $\alpha_J(s, s_{ik})$ are the smooth functions of s_{ik} as compared with the singular part of the amplitude, hence it can be expanded in a series in the singular point and only the first term of this series should be employed further. As s_0 it is convenient to take the middle point of physical region of Dalitz-plot in which $z = 0$. In this case we get $s_{ik} = s_0 = \frac{s+m_1^2+m_2^2+m_3^2}{m_{12}^2+m_{13}^2+m_{23}^2}$, where $m_{ik} = \frac{m_i+m_k}{2}$. We define $\alpha_J(s, s_{ik})$ and $b_J(s_{ik})$ at the point s_0 . Such a choice of point s_0 allows us to replace integral equations (37) by the algebraic equations for the state $\Sigma_c J^P = \frac{3}{2}^-$ of the (10, 2) multiplet:

$$\left\{ \begin{array}{l} \alpha_1^0(s, s_0) = \lambda + \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} + \frac{3}{2} \alpha_0^{1c}(s, s_0) I_{1+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1+}(s_0)} \quad 1+ \\ \alpha_1^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^+1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{1_c^+}(s_0)} - \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1_c^+1_c^+}(s, s_0) \quad 1_c^+ \\ \quad + \frac{3}{2} \alpha_0^{1c}(s, s_0) I_{1_c^+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1_c^+}(s_0)} \\ \alpha_0^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^-1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{1_c^-}(s_0)} + \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1_c^-}(s_0)} \quad 1_c^- \\ \quad + \frac{1}{2} \alpha_0^{1c}(s, s_0) I_{1_c^-1_c^-}(s, s_0) . \end{array} \right. \quad (39)$$

Here the reduced amplitudes for the diquarks 1^+ , 1_c^+ , 1_c^- are given. The function $I_{J_1 J_2}(s, s_0)$ takes into account singularity which corresponds to the simultaneous vanishing of all propagators in the triangle diagrams.

$$I_{J_1 J_2}(s, s_0) = \int_{(m_i+m_k)^2}^{\Lambda_{J_1}} \frac{ds'_{ik}}{\pi} \frac{\rho_{J_1}(s'_{ik}) G_{J_1}^2(s'_{ik})}{s'_{ik} - s_{ik}} \int_{-1}^1 \frac{dz}{2} \frac{1}{1 - B_{J_2}(s_{ij})}. \quad (40)$$

The $G_J(s_{ik})$ functions have the smooth dependence from energy s_{ik} [18] therefore we suggest them as constants. The parameters of model: λ_J cutoff parameter, g_J vertex constants are chosen dimensionless:

$$g_J = \frac{m_i + m_k}{2\pi} G_J, \quad \lambda_J = \frac{4\Lambda_J}{(m_i + m_k)^2}. \quad (41)$$

Here m_i and m_k are quark masses in the intermediate state of the quark loop. We calculate the system equations and can determine the mass values of the $\Sigma_c J^P = \frac{3}{2}^-$ (10, 2). We calculate a pole in s which corresponds to the bound state of three quarks.

By analogy with Σ_c -hyperon we obtain the system equations for the reduced amplitudes for all particles (70, 1^-) multiplets.

5. Calculation results.

The quark masses ($m_u = m_d = m$ and m_c) are not fixed. In any way we assume $m = 570 \text{ MeV}$ and $m_c = 1900 \text{ MeV}$. The value of nonstrange mass m is similar to the our paper ones [15]. In our model the four parameters are used: gluon coupling constants $g_c = 0.85$ and $g_u = 0.58$ and cutoff energy parameters $\lambda_c = 9.2$, $\lambda_u = 10.2$ ($\lambda_{cu} = \frac{1}{4}(\sqrt{\lambda_u} + \sqrt{\lambda_c})^2$) for the charmed and nonstrange diquarks. The parameters have been determined by the baryon masses: $M_{\Lambda_c \frac{1}{2}^-(8,4)} = 2880 \text{ MeV}$, $M_{\Lambda_c \frac{3}{2}^-(8,4)} = 2625 \text{ MeV}$, $M_{\Lambda_c \frac{5}{2}^-(8,4)} = 2765 \text{ MeV}$, and $M_{\Xi_{cc} \frac{5}{2}^-(8,4)} = 3519 \text{ MeV}$. In the Tables I-IV we represent the masses of the charmed resonances belonging to the $(70, 1^-)$ multiplet obtained using the fit of experimental values [21].

The $(70, 1^-)$ charmed baryon multiplet has 23 baryons with different masses. The 6 resonances are in good agreement with the experimental data [21]. We have predicted 17 masses of charmed excited baryons.

In the framework of the proposed approximate method of solving the relativistic three-particle problem, we have obtained a satisfactory spectrum of P -wave charmed baryons.

6. Conclusion.

In strongly bound systems of light and heavy quarks, such as the charmed baryons considered, where $p/m \sim 1$ for the light quarks, the approximation by nonrelativistic kinematics and dynamics is not justified. The relativized quark model applied to baryon spectroscopy by Capstick and Isgur [22].

In the papers [13, 14] the relativistic generalization of Faddeev equations in the framework of dispersion relations are constructed. We calculated the S -wave baryon masses using the method based on the extraction of leading singularities of the amplitude. The behavior of electromagnetic form factor of the nucleon and hyperon in the region of low and intermediate momentum transfers is determined by [23]. In the framework of the dispersion relation approach the charge radii of S -wave baryon multiplets with $J^P = \frac{1}{2}^+$ are calculated.

In our paper [24] the relativistic Faddeev equations for the S -wave charmed baryons are constructed. We calculated the mass spectra of single, double and triple charm baryons using the input four-fermion interaction with quantum numbers of the gluon.

In the framework of a relativistically covariant constituent quark model one calculated on the basic of the Bethe-Salpeter equation in its instantaneous approximation mass spectrum of P -wave charmed baryons [25].

In our paper [15] the relativistic description of three particles amplitudes of P -wave baryons are considered. We take into account the u, d, s -quarks. The mass spectrum of nonstrange and strange states of multiplet $(70, 1^-)$ are calculated. We use only four parameters for the calculation of 30 baryon masses. We take into account the mass shift of u, d, s quarks which allows us to obtain the P -wave baryon bound states [15]. Recently, the mass spectrum baryons of $(70, 1^-)$ multiplet using $1/N_c$ expansion are calculated [26]. The authors solved the problem by removing the splitting of generators and using orbital-flavor-spin wave functions.

We also use the orbital-flavor-spin wave functions for the construction of integral equations. It allows as to calculate the mass spectra for all charmed baryons $(70, 1^-)$ multiplet. The important problem is the mixing of P -wave baryons and the five quark systems (cryptoexotic baryons) [27] and hybrid baryons [28]. We can see that the

masses of P -wave charmed baryons with $J^P = \frac{1}{2}^-$ are heavier than the masses of states with $J^P = \frac{3}{2}^-$ and $J^P = \frac{5}{2}^-$. This conclusion contradicts to the result of nonrelativistic quark models [29 – 32]. The exceptions are the masses of lowest Λ_c -baryons with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$. The lowest state $\Lambda_c(1, 2) J^P = \frac{1}{2}^- (70, 1^-)$ mass is equal to $M = 2400 \text{ MeV}$.

Acknowledgment

The authors would like to thank T. Barnes, S. Capstick, S. Chekanov, Fl. Stancu for useful discussions. The work was carried with the support of the Russian Ministry of Education (grant 2.1.1.68.26).

Table I.

The Σ_c -hyperon masses of multiplet $(70, 1^-)$.

| Multiplet | Baryon | Mass (GeV) | Mass (GeV) (exp.) |
|-------------------------|----------|----------------|-----------------------|
| $\frac{3}{2}^- (10, 2)$ | D_{13} | 2.570 | – |
| $\frac{1}{2}^- (10, 2)$ | S_{11} | 2.915 | – |
| $\frac{5}{2}^- (8, 4)$ | D_{15} | 2.740 | 2.800 |
| $\frac{3}{2}^- (8, 4)$ | D_{13} | 2.570 | – |
| $\frac{1}{2}^- (8, 4)$ | S_{11} | 2.915 | – |
| $\frac{3}{2}^- (8, 2)$ | D_{13} | 2.575 | – |
| $\frac{1}{2}^- (8, 2)$ | S_{11} | 2.700 | – |

The parameters of model (Tables I-IV): gluon coupling constants $g_c = 0.85$, $g_u = 0.58$, cutoff energy parameters $\lambda_c = 9.2$, $\lambda_u = 10.4$.

Table II.

The Λ_c -hyperon masses of multiplet $(70, 1^-)$.

| Multiplet | Baryon | Mass (GeV) | Mass (GeV) (exp.) |
|------------------------|----------|----------------|-----------------------|
| $\frac{5}{2}^- (8, 4)$ | D_{05} | 2.765 | 2.765 |
| $\frac{3}{2}^- (8, 4)$ | D_{03} | 2.625 | 2.625 |
| $\frac{1}{2}^- (8, 4)$ | S_{01} | 2.880 | 2.880 |
| $\frac{3}{2}^- (8, 2)$ | D_{03} | 2.630 | – |
| $\frac{1}{2}^- (8, 2)$ | S_{01} | 2.635 | 2.595 |
| $\frac{3}{2}^- (1, 2)$ | D_{03} | 2.630 | – |
| $\frac{1}{2}^- (1, 2)$ | S_{01} | 2.400 | – |

Table III.

The Ξ_{cc} -hyperon masses of multiplet $(70, 1^-)$.

| Multiplet | Baryon | Mass (GeV) | Mass (GeV) (exp.) |
|-------------------------|----------|----------------|-----------------------|
| $\frac{3}{2}^- (10, 2)$ | D_{13} | 3.140 | — |
| $\frac{1}{2}^- (10, 2)$ | S_{11} | 3.850 | — |
| $\frac{5}{2}^- (8, 4)$ | D_{15} | 3.519 | 3.519 |
| $\frac{3}{2}^- (8, 4)$ | D_{13} | 3.140 | — |
| $\frac{1}{2}^- (8, 4)$ | S_{11} | 3.850 | — |
| $\frac{3}{2}^- (8, 2)$ | D_{13} | 3.240 | — |
| $\frac{1}{2}^- (8, 2)$ | S_{11} | 3.410 | — |

Table IV.

The Ω_{ccc} -hyperon masses of multiplet $(70, 1^-)$.

| Multiplet | Baryon | Mass (GeV) | Mass (GeV) (exp.) |
|-------------------------|----------|----------------|-----------------------|
| $\frac{3}{2}^- (10, 2)$ | D_{03} | 3.470 | — |
| $\frac{1}{2}^- (10, 2)$ | S_{01} | 4.585 | — |

Table V. Coefficient of Ghew-Mandelstam functions for the different diquarks.

| | α_J | β_J | δ_J |
|-------|----------------|--|-----------------------------|
| 1^+ | $\frac{1}{3}$ | $\frac{4m_i m_k}{3(m_i + m_k)^2} - \frac{1}{6}$ | $-\frac{1}{6}(m_i - m_k)^2$ |
| 0^+ | $\frac{1}{2}$ | $-\frac{1}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$ | 0 |
| 0^- | 0 | $\frac{1}{2}$ | $-\frac{1}{2}(m_i - m_k)^2$ |
| 1^- | $\frac{1}{2}$ | $-\frac{1}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2}$ | 0 |
| 2^- | $\frac{3}{10}$ | $\frac{1}{5} \left(1 - \frac{3}{2} \frac{(m_i - m_k)^2}{(m_i + m_k)^2} \right)$ | $-\frac{1}{5}(m_i - m_k)^2$ |

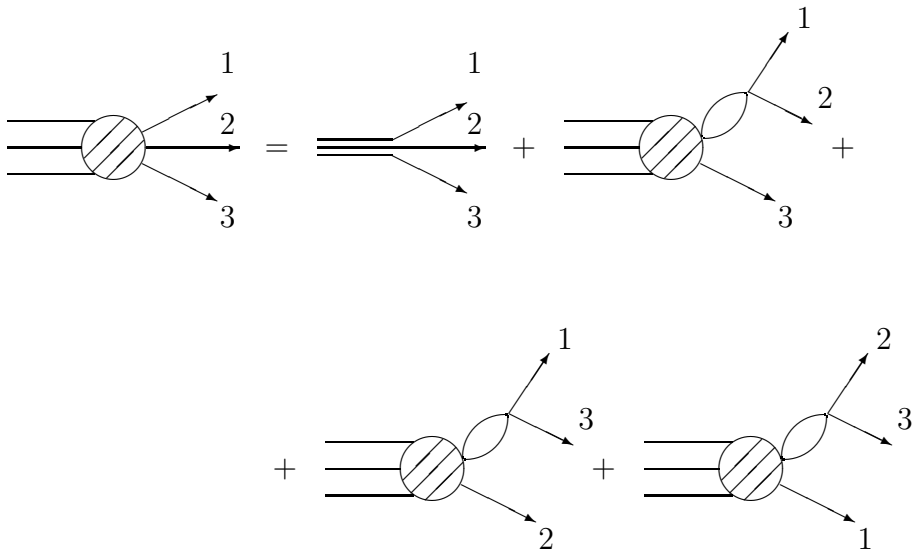


Fig.1. The contribution of diagrams at the last pair of the interacting particles.

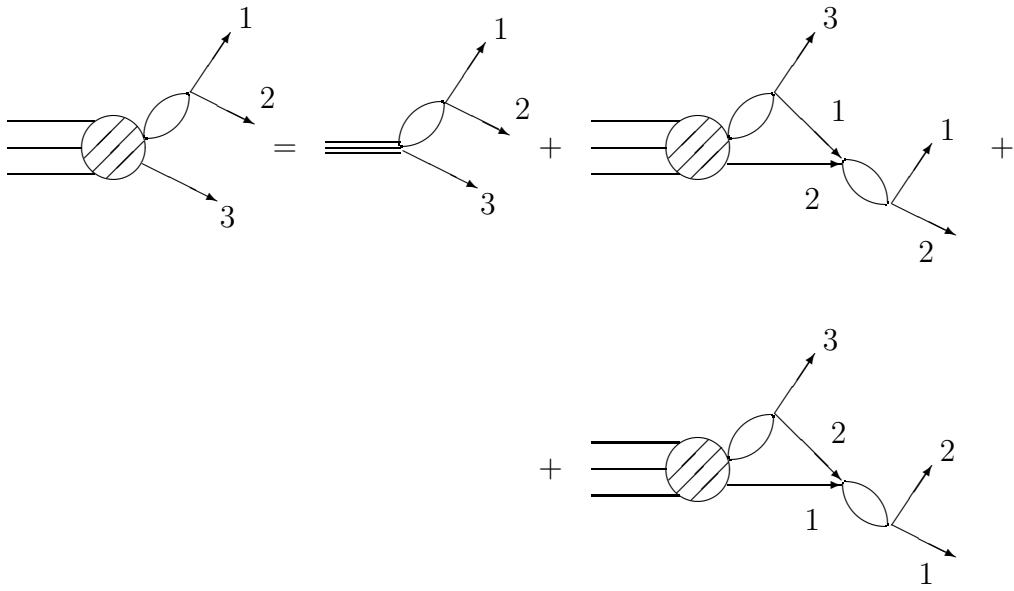


Fig.2. Graphic representation of the equations for the amplitude $A_1(s, s_{ik})$.

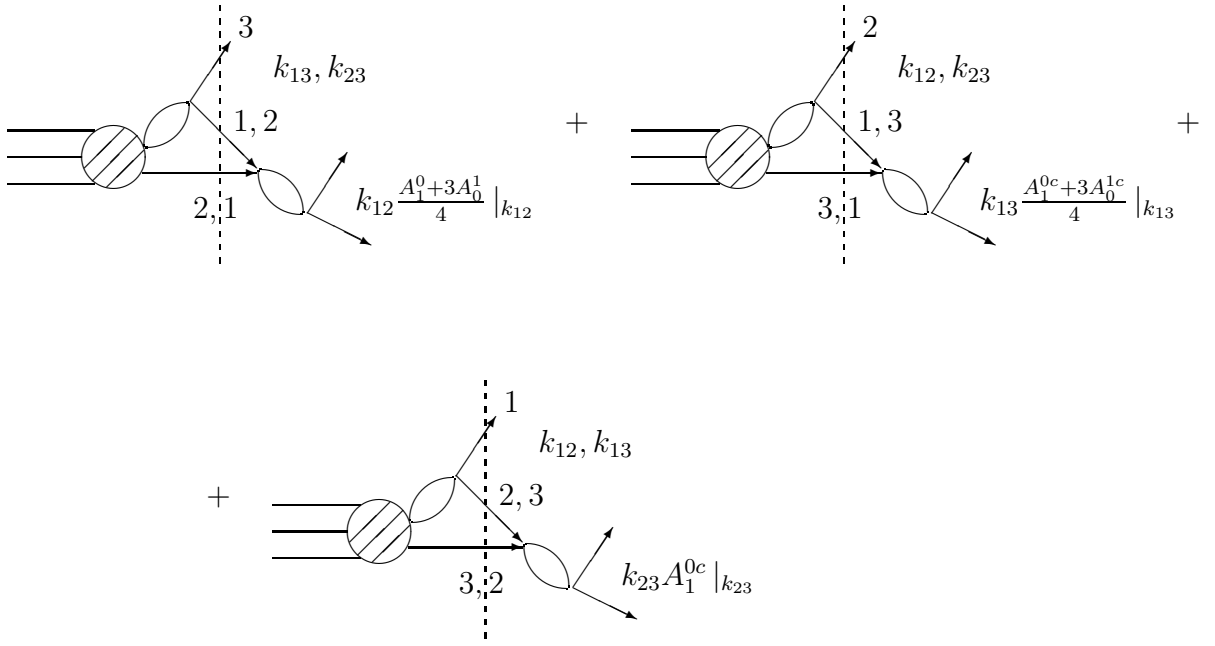


Fig.3. The contribution of the diagrams with the rescattering.

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Appendix A. The P-wave baryon wave functions.

The wave functions of (10, 2) decuplet.

We considered this decuplet in the Section 2. The totally symmetric $SU(6) \times O(3)$ wave function for each decuplet particle has the following form:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right) = \frac{1}{\sqrt{2}} \varphi_S^{SU(3)} \left(\varphi_{MA}^{SU(2)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(2)} \varphi_{MS}^{O(3)} \right). \quad (A1)$$

The functions $\varphi_{MA}^{SU(2)}$, $\varphi_{MS}^{SU(2)}$, $\varphi_{MA}^{O(3)}$, $\varphi_{MS}^{O(3)}$ are:

$$\varphi_{MA}^{SU(2)} = \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \varphi_{MS}^{SU(2)} = \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow), \quad (A2)$$

$$\varphi_{MA}^{O(3)} = \frac{1}{\sqrt{2}} (010 - 100), \quad \varphi_{MS}^{O(3)} = \frac{1}{\sqrt{6}} (010 + 100 - 2 \cdot 001). \quad (A3)$$

For the Σ_c^+ -hyperon $SU(3)$ -function is:

$$\varphi_S^{SU(3)} = \frac{1}{\sqrt{3}} (ucu + cuu + uuc). \quad (A4)$$

Then one obtain the $SU(6) \times O(3)$ -function of Σ_c the (10, 2) multiplet:

$$\begin{aligned} \varphi_{\Sigma_c^+(10,2)} = & \frac{\sqrt{6}}{18} \left(2\{u^1 \downarrow u \uparrow c \uparrow\} + \{c^1 \downarrow u \uparrow u \uparrow\} - \right. \\ & \left. - \{u^1 \uparrow u \downarrow c \uparrow\} - \{u^1 \uparrow u \uparrow c \downarrow\} - \{c^1 \uparrow u \uparrow u \downarrow\} \right). \end{aligned} \quad (A5)$$

The replacement by $u \leftrightarrow c$ or $d \leftrightarrow c$ allows us to obtain the Ξ_{cc} wave function using the Σ_c wave function. In the case of Ω_{ccc}^- the $SU(6) \times O(3)$ wave functions are given:

$$\varphi_{\Omega_{ccc}^-(10,2)} = \frac{\sqrt{2}}{6} \left(\{c^1 \downarrow c \uparrow c \uparrow\} - \{c^1 \uparrow c \uparrow c \downarrow\} \right). \quad (A6)$$

The wave functions of (8, 2) octet.

The wave functions of octet $\frac{3}{2}^-, \frac{1}{2}^-$ (8, 2) multiplet are constructed as:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right), \quad (A7)$$

here

$$\varphi_{MA}^{SU(6)} = \frac{1}{\sqrt{2}} \left(\varphi_{MS}^{SU(3)} \varphi_{MA}^{SU(2)} + \varphi_{MA}^{SU(3)} \varphi_{MS}^{SU(2)} \right), \quad (A8)$$

$$\varphi_{MS}^{SU(6)} = \frac{1}{\sqrt{2}} \left(-\varphi_{MS}^{SU(3)} \varphi_{MS}^{SU(2)} + \varphi_{MA}^{SU(3)} \varphi_{MA}^{SU(2)} \right). \quad (A9)$$

In the case of Σ_c^+ the $SU(3)$ wave functions $\varphi_{MS}^{SU(3)}$ and $\varphi_{MA}^{SU(3)}$ have the following form:

$$\varphi_{MS}^{SU(3)} = \frac{1}{\sqrt{6}}(ucu + cuu - 2wuc), \quad \varphi_{MA}^{SU(3)} = \frac{1}{\sqrt{2}}(ucu - cuu). \quad (A10)$$

Then we can obtain the symmetric wave function for Σ_c^+ :

$$\begin{aligned} \varphi_{\Sigma_c^+(8,2)} = \frac{\sqrt{6}}{18} & \left(2\{u^1 \uparrow u \downarrow c \uparrow\} + \{c^1 \downarrow u \uparrow u \uparrow\} - \{u^1 \uparrow u \uparrow c \downarrow\} - \right. \\ & \left. - \{u^1 \downarrow u \uparrow c \uparrow\} - \{c^1 \uparrow u \uparrow u \downarrow\} \right). \end{aligned} \quad (A11)$$

The Ξ_{cc}^0 wave functions are obtained with replacement by $u \leftrightarrow c$ in Σ_c^+ .

The Λ_c^0 $SU(3)$ wave functions $\varphi_{MS}^{SU(3)}$ and $\varphi_{MA}^{SU(3)}$ are given:

$$\varphi_{MS}^{SU(3)} = \frac{1}{2}(dcu - ucd + cdu - cud), \quad (A12)$$

$$\varphi_{MA}^{SU(3)} = \frac{\sqrt{3}}{6}(cdu - cud + ucd - dcu - 2duc + 2udc). \quad (A13)$$

Then the symmetric $SU(6) \times O(3)$ wave function for $\Lambda_c^0 \frac{3}{2}^-, \frac{1}{2}^-$ can be considered as:

$$\begin{aligned} \varphi_{\Lambda_c^0(8,2)} = \frac{1}{6} & \left(\{u^1 \uparrow d \uparrow c \downarrow\} - \{u^1 \downarrow d \uparrow c \uparrow\} - \{d^1 \uparrow u \uparrow c \downarrow\} + \right. \\ & \left. + \{d^1 \downarrow u \uparrow c \uparrow\} - \{c^1 \uparrow u \uparrow d \downarrow\} + \{c^1 \uparrow u \downarrow d \uparrow\} \right). \end{aligned} \quad (A14)$$

The wave function of (8, 4) octet.

By analogy with the cases (10, 2) and (8, 2) we can calculate the (8, 4) octet wave functions:

$$\varphi = \frac{1}{\sqrt{2}} \left(\varphi_{MA}^{SU(6)} \varphi_{MA}^{O(3)} + \varphi_{MS}^{SU(6)} \varphi_{MS}^{O(3)} \right), \quad (A15)$$

here

$$\varphi_{MA}^{SU(6)} = \varphi_{MA}^{SU(3)} \varphi_S^{SU(2)}, \quad \varphi_{MS}^{SU(6)} = \varphi_{MS}^{SU(3)} \varphi_S^{SU(2)}. \quad (A16)$$

$SU(2)$ wave function is totally symmetric:

$$\varphi_S^{SU(2)} = \uparrow\uparrow\uparrow, \quad (A17)$$

$\varphi_{MS}^{SU(3)}$ and $\varphi_{MA}^{SU(3)}$ similar to one of the (8, 2) multiplet.

For the $\Sigma_c^+ \frac{3}{2}^-, \frac{1}{2}^-$ of (8, 4) multiplet one have:

$$\varphi_{\Sigma_c^+(8,4)} = \frac{\sqrt{2}}{6} \left(\{c^1 \uparrow u \uparrow u \uparrow\} - \{u^1 \uparrow u \uparrow c \uparrow\} \right). \quad (A18)$$

For the Ξ_{cc}^0 we can replace by $u \leftrightarrow c$ in Σ_c^+ .

We obtain the wave function of the Λ_c^0 (8, 4):

$$\varphi_{\Lambda_c^0(8,4)} = \frac{\sqrt{3}}{6} \left(-\{u^1 \uparrow d \uparrow c \uparrow\} + \{d^1 \uparrow u \uparrow c \uparrow\} \right). \quad (A19)$$

The (1, 2) singlet wave function.

We can use the totally symmetric $SU(6) \times O(3)$ wave function in the form:

$$\varphi = \varphi_A^{SU(3)} \varphi_A^{SU(2) \times O(3)}, \quad (A20)$$

here

$$\varphi_A^{SU(3)} = \frac{1}{\sqrt{6}} (cdu - cud + ucd - dcu + duc - udc), \quad (A21)$$

$$\varphi_A^{SU(2) \times O(3)} = \frac{1}{\sqrt{2}} (\varphi_{MS}^{SU(2)} \varphi_{MA}^{O(3)} - \varphi_{MA}^{SU(2)} \varphi_{MS}^{O(3)}). \quad (A22)$$

As result we obtain:

$$\begin{aligned} \varphi_{\Lambda_c^0(1,2)} = \frac{\sqrt{3}}{6} & \left(-\{u^1 \uparrow d \uparrow c \downarrow\} + \{u^1 \uparrow d \downarrow c \uparrow\} + \{d^1 \uparrow u \uparrow c \downarrow\} - \right. \\ & \left. - \{d^1 \uparrow u \downarrow c \uparrow\} - \{c^1 \uparrow u \uparrow d \downarrow\} + \{c^1 \uparrow u \downarrow d \uparrow\} \right). \end{aligned} \quad (A23)$$

Appendix B. The integral equations for the $(70, 1^-)$ multiplet.

The (10, 2) multiplet.

We can represent the equations for the $\frac{3}{2}^- (10, 2)$ multiplet, which is determined by the projection of orbital angular momentum $l_z = +1$. We consider the following states: $\Sigma_c \frac{3}{2}^-$:

$$\left\{ \begin{aligned} A_1^0(s, s_{12}) &= \lambda b_{1+}(s_{12}) L_{1+}(s_{12}) + K_{1+}(s_{12}) \left[\frac{1}{4} A_1^{0c}(s, s_{13}) + \frac{3}{4} A_1^{0c}(s, s_{13}) + \right. \\ &\quad \left. + \frac{1}{4} A_1^{0c}(s, s_{23}) + \frac{3}{4} A_0^{1c}(s, s_{23}) \right] \\ A_1^{0c}(s, s_{13}) &= \lambda b_{1+}(s_{13}) L_{1+}(s_{13}) + K_{1+}(s_{13}) \left[\frac{1}{2} A_1^0(s, s_{12}) - \frac{1}{4} A_1^{0c}(s, s_{12}) + \right. \\ &\quad \left. + \frac{3}{4} A_0^{1c}(s, s_{12}) + \frac{1}{2} A_1^0(s, s_{23}) - \frac{1}{4} A_1^{0c}(s, s_{23}) + \frac{3}{4} A_0^{1c}(s, s_{23}) \right] \\ A_0^{1c}(s, s_{23}) &= \lambda b_{1-}(s_{23}) L_{1-}(s_{23}) + K_{1-}(s_{23}) \left[\frac{1}{2} A_1^0(s, s_{12}) + \frac{1}{4} A_1^{0c}(s, s_{12}) + \right. \\ &\quad \left. + \frac{1}{4} A_0^{1c}(s, s_{12}) + \frac{1}{2} A_1^0(s, s_{13}) + \frac{1}{4} A_1^{0c}(s, s_{13}) + \frac{1}{4} A_0^{1c}(s, s_{13}) \right]. \end{aligned} \right. \quad (A24)$$

$\Omega_{ccc} \frac{3}{2}^-$:

$$\left\{ \begin{aligned} A_1^{0cc}(s, s_{12}) &= \lambda b_{1_{cc}^+}(s_{12}) L_{1_{cc}^+}(s_{12}) + K_{1_{cc}^+}(s_{12}) \left[\frac{1}{4} A_1^{0cc}(s, s_{13}) + \frac{3}{4} A_0^{1cc}(s, s_{13}) + \right. \\ &\quad \left. + \frac{1}{4} A_1^{0cc}(s, s_{23}) + \frac{3}{4} A_0^{1cc}(s, s_{23}) \right] \\ A_0^{1cc}(s, s_{13}) &= \lambda b_{1_{cc}^-}(s_{13}) L_{1_{cc}^-}(s_{13}) + K_{1_{cc}^-}(s_{13}) \left[\frac{3}{4} A_1^{0cc}(s, s_{12}) + \frac{1}{4} A_0^{1cc}(s, s_{12}) + \right. \\ &\quad \left. + \frac{3}{4} A_1^{0cc}(s, s_{23}) + \frac{1}{4} A_0^{1cc}(s, s_{23}) \right]. \end{aligned} \right. \quad (A25)$$

The system integral equations for the $\Xi_{cc} \frac{3}{2}^-$ are similar to $\Sigma_c \frac{3}{2}^-$ with the replacement by $u \leftrightarrow c$.

The (8, 2) multiplet.

We determine the equations of multiplet $\frac{3}{2}^- (8, 2)$.

$\Sigma_c \frac{3}{2}^-$:

$$\left\{ \begin{array}{l} A_1^0(s, s_{12}) = \lambda b_{1+}(s_{12})L_{1+}(s_{12}) + K_{1+}(s_{12}) \left[-\frac{1}{8}A_1^{0c}(s, s_{13}) + \frac{3}{8}A_0^{1c}(s, s_{13}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{0c}(s, s_{13}) + \frac{3}{8}A_1^{1c}(s, s_{13}) - \frac{1}{8}A_1^{0c}(s, s_{23}) + \frac{3}{8}A_0^{1c}(s, s_{23}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{0c}(s, s_{23}) + \frac{3}{8}A_1^{1c}(s, s_{23}) \right] \\ A_1^{0c}(s, s_{13}) = \lambda b_{1c}^+(s_{13})L_{1c}^+(s_{13}) + K_{1c}^+(s_{13}) \left[\frac{1}{2}A_1^0(s, s_{12}) - \frac{5}{8}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{1c}(s, s_{12}) + \frac{3}{8}A_0^{0c}(s, s_{12}) + \frac{3}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^0(s, s_{23}) - \right. \\ \quad \left. - \frac{5}{8}A_1^{0c}(s, s_{23}) + \frac{3}{8}A_0^{1c}(s, s_{23}) + \frac{3}{8}A_0^{0c}(s, s_{23}) + \frac{3}{8}A_1^{1c}(s, s_{23}) \right] \\ A_0^{1c}(s, s_{23}) = \lambda b_{1c}^-(s_{23})L_{1c}^-(s_{23}) + K_{1c}^-(s_{23}) \left[\frac{1}{2}A_1^0(s, s_{12}) - \frac{1}{8}A_1^{0c}(s, s_{12}) - \right. \\ \quad \left. - \frac{1}{8}A_0^{1c}(s, s_{12}) + \frac{3}{8}A_0^{0c}(s, s_{12}) + \frac{3}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^0(s, s_{13}) - \right. \\ \quad \left. - \frac{1}{8}A_1^{0c}(s, s_{13}) - \frac{1}{8}A_0^{1c}(s, s_{13}) + \frac{3}{8}A_0^{0c}(s, s_{13}) + \frac{3}{8}A_1^{1c}(s, s_{13}) \right] \\ A_0^{0c}(s, s_{13}) = \lambda b_{0c}^+(s_{13})L_{0c}^+(s_{13}) + K_{0c}^+(s_{13}) \left[\frac{1}{2}A_1^0(s, s_{12}) - \frac{1}{8}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{1c}(s, s_{12}) - \frac{1}{8}A_0^{0c}(s, s_{12}) + \frac{3}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^0(s, s_{23}) - \right. \\ \quad \left. - \frac{1}{8}A_1^{0c}(s, s_{23}) + \frac{3}{8}A_0^{1c}(s, s_{23}) - \frac{1}{8}A_0^{0c}(s, s_{23}) + \frac{3}{8}A_1^{1c}(s, s_{23}) \right] \\ A_1^{1c}(s, s_{23}) = \lambda b_{2c}^-(s_{23})L_{2c}^-(s_{23}) + K_{2c}^-(s_{23}) \left[\frac{1}{2}A_1^0(s, s_{12}) - \frac{1}{8}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{1c}(s, s_{12}) + \frac{3}{8}A_0^{0c}(s, s_{12}) - \frac{1}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^0(s, s_{13}) - \right. \\ \quad \left. - \frac{1}{8}A_1^{0c}(s, s_{13}) + \frac{3}{8}A_0^{1c}(s, s_{13}) + \frac{3}{8}A_0^{0c}(s, s_{13}) - \frac{1}{8}A_1^{1c}(s, s_{13}) \right] . \end{array} \right. \quad (A26)$$

$\Lambda_c \frac{3}{2}^-$:

$$\left\{ \begin{array}{l} A_1^1(s, s_{12}) = \lambda b_{1-}(s_{12})L_{1-}(s_{12}) + K_{1-}(s_{12}) \left[\frac{3}{8}A_1^{0c}(s, s_{13}) - \frac{1}{8}A_0^{1c}(s, s_{13}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{0c}(s, s_{13}) + \frac{3}{8}A_1^{1c}(s, s_{13}) + \frac{3}{8}A_1^{0c}(s, s_{23}) - \frac{1}{8}A_0^{1c}(s, s_{23}) + \right. \\ \quad \left. + \frac{3}{8}A_0^{0c}(s, s_{23}) + \frac{3}{8}A_1^{1c}(s, s_{23}) \right] \\ A_1^{0c}(s, s_{13}) = \lambda b_{1c}^+(s_{13})L_{1c}^+(s_{13}) + K_{1c}^+(s_{13}) \left[\frac{1}{2}A_1^1(s, s_{12}) - \frac{1}{8}A_1^{0c}(s, s_{12}) - \right. \\ \quad \left. - \frac{1}{8}A_0^{1c}(s, s_{12}) + \frac{3}{8}A_0^{0c}(s, s_{12}) + \frac{3}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^1(s, s_{23}) - \right. \\ \quad \left. - \frac{1}{8}A_1^{0c}(s, s_{23}) - \frac{1}{8}A_0^{1c}(s, s_{23}) + \frac{3}{8}A_0^{0c}(s, s_{23}) + \frac{3}{8}A_1^{1c}(s, s_{23}) \right] \\ A_0^{1c}(s, s_{23}) = \lambda b_{1c}^-(s_{23})L_{1c}^-(s_{23}) + K_{1c}^-(s_{23}) \left[\frac{1}{2}A_1^1(s, s_{12}) + \frac{3}{8}A_1^{0c}(s, s_{12}) - \right. \\ \quad \left. - \frac{5}{8}A_0^{1c}(s, s_{12}) + \frac{3}{8}A_0^{0c}(s, s_{12}) + \frac{3}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^1(s, s_{13}) + \right. \\ \quad \left. + \frac{3}{8}A_1^{0c}(s, s_{13}) - \frac{5}{8}A_0^{1c}(s, s_{13}) + \frac{3}{8}A_0^{0c}(s, s_{13}) + \frac{3}{8}A_1^{1c}(s, s_{13}) \right] \\ A_0^{0c}(s, s_{13}) = \lambda b_{0c}^+(s_{13})L_{0c}^+(s_{13}) + K_{0c}^+(s_{13}) \left[\frac{1}{2}A_1^1(s, s_{12}) + \frac{3}{8}A_1^{0c}(s, s_{12}) - \right. \\ \quad \left. - \frac{1}{8}A_0^{1c}(s, s_{12}) - \frac{1}{8}A_0^{0c}(s, s_{12}) + \frac{3}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^1(s, s_{23}) + \right. \\ \quad \left. + \frac{3}{8}A_1^{0c}(s, s_{23}) - \frac{1}{8}A_0^{1c}(s, s_{23}) - \frac{1}{8}A_0^{0c}(s, s_{23}) + \frac{3}{8}A_1^{1c}(s, s_{23}) \right] \\ A_1^{1c}(s, s_{23}) = \lambda b_{2c}^-(s_{23})L_{2c}^-(s_{23}) + K_{2c}^-(s_{23}) \left[\frac{1}{2}A_1^1(s, s_{12}) + \frac{3}{8}A_1^{0c}(s, s_{12}) - \right. \\ \quad \left. - \frac{1}{8}A_0^{1c}(s, s_{12}) + \frac{3}{8}A_0^{0c}(s, s_{12}) - \frac{1}{8}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^1(s, s_{13}) + \right. \\ \quad \left. + \frac{3}{8}A_1^{0c}(s, s_{13}) - \frac{1}{8}A_0^{1c}(s, s_{13}) + \frac{3}{8}A_0^{0c}(s, s_{13}) - \frac{1}{8}A_1^{1c}(s, s_{13}) \right] . \end{array} \right. \quad (A27)$$

The system equations of the $\Xi_{cc} \frac{3}{2}^- (8, 2)$ are similar to the case $\Sigma_c \frac{3}{2}^- (8, 2)$ by replacement $u \leftrightarrow c$.

The (8, 4) multiplet.

We consider the states:

$\Sigma_c \frac{5}{2}^-$:

$$\left\{ \begin{array}{l} A_1^0(s, s_{12}) = \lambda b_{1+}(s_{12})L_{1+}(s_{12}) + K_{1+}(s_{12}) \left[\frac{1}{4}A_1^{0c}(s, s_{13}) + \frac{3}{4}A_1^{1c}(s, s_{13}) + \right. \\ \quad \left. + \frac{1}{4}A_1^{0c}(s, s_{23}) + \frac{3}{4}A_1^{1c}(s, s_{23}) \right] \\ A_1^{0c}(s, s_{13}) = \lambda b_{1c^+}(s_{13})L_{1c^+}(s_{13}) + K_{1c^+}(s_{13}) \left[\frac{1}{2}A_1^0(s, s_{12}) - \frac{1}{4}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{3}{4}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^0(s, s_{23}) - \frac{1}{4}A_1^{0c}(s, s_{23}) + \frac{3}{4}A_1^{1c}(s, s_{23}) \right] \\ A_1^{1c}(s, s_{23}) = \lambda b_{2c^-}(s_{23})L_{2c^-}(s_{23}) + K_{2c^-}(s_{23}) \left[\frac{1}{2}A_1^0(s, s_{12}) + \frac{1}{4}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{1}{4}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^0(s, s_{13}) + \frac{1}{4}A_1^{0c}(s, s_{13}) + \frac{1}{4}A_1^{1c}(s, s_{13}) \right] . \end{array} \right. \quad (A28)$$

$\Lambda_{cc} \frac{5}{2}^-$:

$$\left\{ \begin{array}{l} A_1^1(s, s_{12}) = \lambda b_{2-}(s_{12})L_{2-}(s_{12}) + K_{2-}(s_{12}) \left[\frac{3}{4}A_1^{0c}(s, s_{13}) + \frac{1}{4}A_1^{1c}(s, s_{13}) + \right. \\ \quad \left. + \frac{3}{4}A_1^{0c}(s, s_{23}) + \frac{1}{4}A_1^{1c}(s, s_{23}) \right] \\ A_1^{0c}(s, s_{13}) = \lambda b_{1c^+}(s_{13})L_{1c^+}(s_{13}) + K_{1c^+}(s_{13}) \left[\frac{1}{2}A_1^1(s, s_{12}) + \frac{1}{3}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{1}{6}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_1^1(s, s_{23}) + \frac{1}{3}A_1^{0c}(s, s_{23}) + \frac{1}{6}A_1^{1c}(s, s_{23}) \right] \\ A_1^{1c}(s, s_{23}) = \lambda b_{2c^-}(s_{23})L_{2c^-}(s_{23}) + K_{2c^-}(s_{23}) \left[\frac{1}{2}A_1^1(s, s_{12}) + \frac{1}{2}A_1^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{1}{2}A_1^1(s, s_{13}) + \frac{1}{2}A_1^{0c}(s, s_{13}) \right] . \end{array} \right. \quad (A29)$$

The (1, 2) multiplet.

$\Lambda \frac{3}{2}^-$:

$$\left\{ \begin{array}{l} A_0^0(s, s_{12}) = \lambda b_{0+}(s_{12})L_{0+}(s_{12}) + K_{0+}(s_{12}) \left[\frac{1}{4}A_0^{0c}(s, s_{13}) + \frac{3}{4}A_1^{1c}(s, s_{13}) + \right. \\ \quad \left. + \frac{1}{4}A_0^{0c}(s, s_{23}) + \frac{3}{4}A_1^{1c}(s, s_{23}) \right] \\ A_0^{0c}(s, s_{13}) = \lambda b_{0c^+}(s_{13})L_{0c^+}(s_{13}) + K_{0c^+}(s_{13}) \left[\frac{1}{2}A_0^0(s, s_{12}) - \frac{1}{4}A_0^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{3}{4}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_0^0(s, s_{23}) - \frac{1}{4}A_0^{0c}(s, s_{23}) + \frac{3}{4}A_1^{1c}(s, s_{23}) \right] \\ A_1^{1c}(s, s_{23}) = \lambda b_{2c^-}(s_{23})L_{2c^-}(s_{23}) + K_{2c^-}(s_{23}) \left[\frac{1}{2}A_0^0(s, s_{12}) + \frac{1}{4}A_0^{0c}(s, s_{12}) + \right. \\ \quad \left. + \frac{1}{4}A_1^{1c}(s, s_{12}) + \frac{1}{2}A_0^0(s, s_{13}) + \frac{1}{4}A_0^{0c}(s, s_{13}) + \frac{1}{4}A_1^{1c}(s, s_{13}) \right] . \end{array} \right. \quad (A30)$$

Appendix C. The system equations of reduced amplitude of the multiplets $(70, 1^-)$.

The equations of $(10, 2)$ multiplet.

$\Sigma_c \frac{3}{2}^-$:

$$\left\{ \begin{array}{l} \alpha_1^0(s, s_0) = \lambda + \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} + \frac{3}{2} \alpha_0^{1c}(s, s_0) I_{1+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1+}(s_0)} \quad 1^+ \\ \alpha_1^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^+1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{1_c^+}(s_0)} - \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1_c^+1_c^+}(s, s_0) \quad 1_c^+ \\ \quad + \frac{3}{2} \alpha_0^{1c}(s, s_0) I_{1_c^+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1+}(s_0)} \\ \alpha_0^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^-1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{1_c^-}(s_0)} + \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1_c^-}(s_0)} \quad 1_c^- \\ \quad + \frac{1}{2} \alpha_0^{1c}(s, s_0) I_{1_c^-1_c^-}(s, s_0) . \end{array} \right. \quad (A31)$$

$\Omega_{ccc} \frac{3}{2}^-$:

$$\left\{ \begin{array}{l} \alpha_1^{0cc}(s, s_0) = \lambda + \frac{1}{2} \alpha_1^{0cc}(s, s_0) I_{1_{cc}^+1_{cc}^+}(s, s_0) + \frac{3}{2} \alpha_0^{1cc}(s, s_0) I_{1_{cc}^+1_{cc}^-}(s, s_0) \frac{b_{1_{cc}^-}(s_0)}{b_{1_{cc}^+}(s_0)} \quad 1_{cc}^+ \\ \alpha_0^{1cc}(s, s_0) = \lambda + \frac{3}{2} \alpha_1^{0cc}(s, s_0) I_{1_{cc}^-1_{cc}^+}(s, s_0) \frac{b_{1_{cc}^+}(s_0)}{b_{1_{cc}^-}(s_0)} + \frac{1}{2} \alpha_0^{1cc}(s, s_0) I_{1_{cc}^-1_{cc}^-}(s, s_0) . \quad 1_{cc}^- \end{array} \right. \quad (A32)$$

The Ξ_{cc} system equations is similar to the case Σ_c with the replacement by $u \leftrightarrow c$.

The equations of (8, 2) multiplet.

$\Sigma_c \frac{3}{2}^-$:

$$\left\{ \begin{array}{l}
 \alpha_1^0(s, s_0) = \lambda - \frac{1}{4} \alpha_1^{0c}(s, s_0) I_{1+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} + \frac{3}{4} \alpha_0^{1c}(s, s_0) I_{1+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1+}(s_0)} \\
 \quad + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{1+0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{1+}(s_0)} + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{1+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1+}(s_0)} \\
 \alpha_1^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^+1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{1_c^+}(s_0)} - \frac{5}{4} \alpha_1^{0c}(s, s_0) I_{1_c^+1_c^+}(s, s_0) \\
 \quad + \frac{3}{4} \alpha_0^{1c}(s, s_0) I_{1_c^+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1_c^+}(s_0)} + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{1_c^+0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{1_c^+}(s_0)} \\
 \quad + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{1_c^+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1_c^+}(s_0)} \\
 \alpha_1^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^-1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{1_c^-}(s_0)} - \frac{1}{4} \alpha_1^{0c}(s, s_0) I_{1_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1_c^-}(s_0)} \\
 \quad - \frac{1}{4} \alpha_0^{1c}(s, s_0) I_{1_c^-1_c^-}(s, s_0) + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{1_c^-0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{1_c^-}(s_0)} \\
 \quad + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{1_c^-2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1_c^-}(s_0)} \\
 \alpha_0^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{0_c^+1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{0_c^+}(s_0)} - \frac{1}{4} \alpha_1^{0c}(s, s_0) I_{0_c^+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{0_c^+}(s_0)} \\
 \quad + \frac{3}{4} \alpha_0^{1c}(s, s_0) I_{0_c^+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{0_c^+}(s_0)} - \frac{1}{4} \alpha_0^{0c}(s, s_0) I_{0_c^+0_c^+}(s, s_0) \\
 \quad + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{0_c^+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{0_c^+}(s_0)} \\
 \alpha_1^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{2_c^-1+}(s, s_0) \frac{b_{1+}(s_0)}{b_{2_c^-}(s_0)} - \frac{1}{4} \alpha_1^{0c}(s, s_0) I_{2_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} \\
 \quad + \frac{3}{4} \alpha_0^{1c}(s, s_0) I_{2_c^-1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{2_c^-}(s_0)} + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{2_c^-0_c^+}(s, s_0) \\
 \quad \frac{b_{0_c^+}(s_0)}{b_{2_c^-}(s_0)} - \frac{1}{4} \alpha_1^{1c}(s, s_0) I_{2_c^-2_c^-}(s, s_0) .
 \end{array} \right. \quad (A33)$$

$\Lambda_c \frac{3}{2}^-$:

$$\left\{ \begin{array}{l}
\alpha_1^0(s, s_0) = \lambda + \frac{3}{4} \alpha_1^{0c}(s, s_0) I_{1+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} - \frac{1}{4} \alpha_0^{1c}(s, s_0) I_{1+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1+}(s_0)} \\
\quad + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{1+0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{1+}(s_0)} + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{1+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1+}(s_0)} \\
\alpha_1^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^+1+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} - \frac{1}{4} \alpha_1^{0c}(s, s_0) I_{1_c^+1_c^+}(s, s_0) \\
\quad - \frac{1}{4} \alpha_0^{1c}(s, s_0) I_{1_c^+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{1+}(s_0)} + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{1_c^+0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{1+}(s_0)} \\
\quad + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{1_c^+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1+}(s_0)} \\
\alpha_0^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^-1+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1_c^-}(s_0)} + \frac{3}{4} \alpha_1^{0c}(s, s_0) I_{1_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1_c^-}(s_0)} \\
\quad - \frac{5}{4} \alpha_0^{1c}(s, s_0) I_{1_c^-1_c^-}(s, s_0) + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{1_c^-0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{1_c^-}(s_0)} \\
\quad + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{1_c^-2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1_c^-}(s_0)} \\
\alpha_0^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{0_c^+1+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{0_c^+}(s_0)} + \frac{3}{4} \alpha_1^{0c}(s, s_0) I_{0_c^+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{0_c^+}(s_0)} \\
\quad - \frac{1}{4} \alpha_0^{1c}(s, s_0) I_{0_c^+1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{0_c^+}(s_0)} - \frac{1}{4} \alpha_0^{0c}(s, s_0) I_{0_c^+0_c^+}(s, s_0) \\
\quad + \frac{3}{4} \alpha_1^{1c}(s, s_0) I_{0_c^+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{0_c^+}(s_0)} \\
\alpha_1^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{2_c^-1+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} + \frac{3}{4} \alpha_1^{0c}(s, s_0) I_{2_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} \\
\quad - \frac{1}{4} \alpha_0^{1c}(s, s_0) I_{2_c^-1_c^-}(s, s_0) \frac{b_{1_c^-}(s_0)}{b_{2_c^-}(s_0)} + \frac{3}{4} \alpha_0^{0c}(s, s_0) I_{2_c^-0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{2_c^-}(s_0)} \\
\quad - \frac{1}{4} \alpha_1^{1c}(s, s_0) I_{2_c^-2_c^-}(s, s_0) .
\end{array} \right. \quad (A34)$$

By analogy with the case (10, 2), the equations for the $\Xi_{cc} \frac{3}{2}^-$ (8, 2) are similar to $\Sigma_c \frac{3}{2}^-$ with replacement by $u \leftrightarrow c$.

The equations of (8, 4) multiplet.

$\Sigma_c \frac{5}{2}^-$:

$$\left\{ \begin{array}{l}
\alpha_1^0(s, s_0) = \lambda + \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1+1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} + \frac{3}{2} \alpha_1^{1c}(s, s_0) I_{1+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1+}(s_0)} \\
\alpha_1^{0c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{1_c^+1+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{1+}(s_0)} - \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{1_c^+1_c^+}(s, s_0) \\
\quad + \frac{3}{2} \alpha_1^{1c}(s, s_0) I_{1_c^+2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1+}(s_0)} \\
\alpha_1^{1c}(s, s_0) = \lambda + \alpha_1^0(s, s_0) I_{2_c^-1+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} + \frac{1}{2} \alpha_1^{0c}(s, s_0) I_{2_c^-1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} \\
\quad + \frac{1}{2} \alpha_1^{1c}(s, s_0) I_{2_c^-2_c^-}(s, s_0) .
\end{array} \right. \quad (A35)$$

$\Lambda_c \frac{5}{2}^-:$

$$\left\{ \begin{array}{l} \alpha_1^1(s, s_0) = \lambda + \frac{3}{2} \alpha_1^{0c}(s, s_0) I_{2^- 1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} + \frac{1}{2} \alpha_1^{1c}(s, s_0) I_{2^- 2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{2_c^-}(s_0)} \quad 2^- \\ \alpha_1^{0c}(s, s_0) = \lambda + \alpha_1^1(s, s_0) I_{1_c^+ 2^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1_c^+}(s_0)} + \frac{2}{3} \alpha_1^{0c}(s, s_0) I_{1_c^+ 1_c^+}(s, s_0) \quad 1_c^+ \\ \quad + \frac{1}{3} \alpha_1^{1c}(s, s_0) I_{1_c^+ 2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{1_c^+}(s_0)} \\ \alpha_1^{1c}(s, s_0) = \lambda + \alpha_1^1(s, s_0) I_{2_c^- 2^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{2_c^-}(s_0)} + \alpha_1^{0c}(s, s_0) I_{2_c^- 1_c^+}(s, s_0) \frac{b_{1_c^+}(s_0)}{b_{2_c^-}(s_0)} . \quad 2_c^- \end{array} \right. \quad (A36)$$

The equations of (1, 2) singlet.

$\Lambda_c \frac{3}{2}^-:$

$$\left\{ \begin{array}{l} \alpha_0^0(s, s_0) = \lambda + \frac{1}{2} \alpha_0^{0c}(s, s_0) I_{0^+ 0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{0^+}(s_0)} + \frac{3}{2} \alpha_1^{1c}(s, s_0) I_{0^+ 2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{0^+}(s_0)} \quad 0^+ \\ \alpha_0^{0c}(s, s_0) = \lambda + \alpha_0^0(s, s_0) I_{0_c^+ 0^+}(s, s_0) \frac{b_{0^+}(s_0)}{b_{0_c^+}(s_0)} - \frac{1}{2} \alpha_0^{0c}(s, s_0) I_{0_c^+ 0_c^+}(s, s_0) \quad 0_c^+ \\ \quad + \frac{3}{2} \alpha_1^{1c}(s, s_0) I_{0_c^+ 2_c^-}(s, s_0) \frac{b_{2_c^-}(s_0)}{b_{0_c^+}(s_0)} \\ \alpha_1^{1c}(s, s_0) = \lambda + \alpha_0^0(s, s_0) I_{2_c^- 0^+}(s, s_0) \frac{b_{0^+}(s_0)}{b_{2_c^-}(s_0)} + \frac{1}{2} \alpha_0^{0c}(s, s_0) I_{2_c^- 0_c^+}(s, s_0) \frac{b_{0_c^+}(s_0)}{b_{2_c^-}(s_0)} \quad 2_c^- \\ \quad + \frac{1}{2} \alpha_1^{1c}(s, s_0) I_{2_c^- 2_c^-}(s, s_0) . \end{array} \right. \quad (A37)$$